Cambridge International AS & A Level

----

https://babacambridgesolutions.com

Mathematics 9709/32

Paper 3 Pure Mathematics 3 **October/November 2023**

Question No (2)

2 The parametric equations of a curve are

$$x = (\ln t)^2, \qquad y = e^{2-t^2},$$

for $t > 0$.

Find the gradient of the curve at the point where $t = e$, simplifying your answer.

Solution:

$x = (\ln t)^2$, $y = e^{2-t^2}$

Diff w.r.t t

$\frac{dx}{dt} = 2(\ln t)\frac{d}{dt}(\ln t)$

$= 2\ln t\left(\frac{1}{t}\right)$

$= \frac{2\ln t}{t}$

$\frac{dy}{dt} = e^{2-t^2}(0-2t)$

$= -2t\,e^{2-t^2}$

using chain rule

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \left(-2t\,e^{2-t^2}\right)\left(\frac{t}{2\ln t}\right)$

$\frac{dy}{dx} = -\frac{t^2 e^{2-t^2}}{\ln t}$

when $t = e$

$\frac{dy}{dx} = -\frac{e^2\,e^{2-e^2}}{\ln e} = -e^{4-e^2}$ $\because \ln e = 1$

$\therefore \frac{dy}{dx} = -e^{4-e^2}$ Ans