Cambridge International AS & A Level

---

https://babacambridgesolutions.com

Mathematics 9709/32

Paper 3 Pure Mathematics 3 **October/November 2022**

Question No (2)

2 The polynomial $2x^3 - x^2 + a$, where a is a constant, is denoted by $\mathrm{p}(x)$. It is given that $(2x + 3)$ is a factor of $\mathrm{p}(x)$.

(a) Find the value of a.

(b) When a has this value, solve the inequality $\mathrm{p}(x) < 0$.

Solution:

(a) $P(x) = 2x^3 - x^2 + a \rightarrow (1)$

Given That $2x+3$ is a factor of $P(x)$

$\therefore P(-3/2) = 0$

$\therefore 2x+3 = 0$

$x = -3/2$

From (1)

$2(-3/2)^3 - (-3/2)^2 + a = 0$

$2\left(-\frac{27}{8}\right) - \frac{9}{4} + a = 0$

$a = +\frac{27}{4} + \frac{9}{4}$

$a = \frac{36}{4} = 9$

(b) $P(2x^3 - x^2 + 9)$

using long division

$$
\begin{array}{r|l}
 & x^2 - 2x + 3 \\
\hline
2x+3 & 2x^3 - x^2 + 9 \\
 & -2x^3 \pm 3x^2 \\
\hline
 & -4x^2 + 9 \\
 & \mp 4x^2 - \quad \pm 6x \\
\hline
 & 6x + 9 \\
 & 6x + 9 \\
 & - \quad - \\
\hline
 & 0
\end{array}
$$

$\therefore P(x) = (2x+3)(x^2 - 2x + 3)$

Given That $P(x) < 0$

$\Rightarrow (2x+3)(x^2-2x+3) < 0$

consider the quadratic factor

x^2-2x+3

using discriminant, $D = b^2-4ac$

$= (-2)^2 - 4(1)(3)$

$= 4-12$

$= -8 < 0$

(no real roots)

$\Rightarrow (x^2-2x+3) > 0$ for all values of x

$\therefore (2x+3) < 0$

$\Rightarrow x < -3/2$