Cambridge International AS & A Level

Mathematics   9709/32

Paper 3 Pure Mathematics 3   **October/November 2020**

Question No (2)

2 **(a)** Expand $\sqrt[3]{1+6x}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients.

(b) State the set of values of x for which the expansion is valid.

Solution:

(a)

Binomial expansion
$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \dfrac{n(n-1)(n-2)^{\square}}{8!}x^3 + \cdots x^n$

$$\sqrt[3]{1+6x} = (1+6x)^{\frac{1}{3}}$$

Applying the binomial theorem up to the term in x^3

$$= 1 + \frac{1}{3}(6x) + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}(6x)^2 + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{8!}(6x)^3 + \cdots$$

$$= 1 + 2x + \frac{\frac{1}{3}\left(-\frac{2}{3}\right)}{2!}36x^2 + \frac{\frac{1}{3}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{6}(216\,x^3) + \cdots.$$

$$= 1 + 2x - \frac{1}{9}(36x^2) + \frac{5}{81}(21(x^3)) + \cdots\Big|$$

$$= 1 + 2x - 4x^2 + \frac{40}{3}x^3 + \cdots \ldots \ldots \ldots. \textbf{Ans}$$

(b)

The expansion will be valid for

$$|6x| < 1$$

$$\Rightarrow -1 < 6x < 1 \qquad \textit{By algebra prop } |a| < 1$$

$$-\frac{1}{6} < x < \frac{1}{6} \qquad\qquad \Rightarrow -1 < a < 1$$