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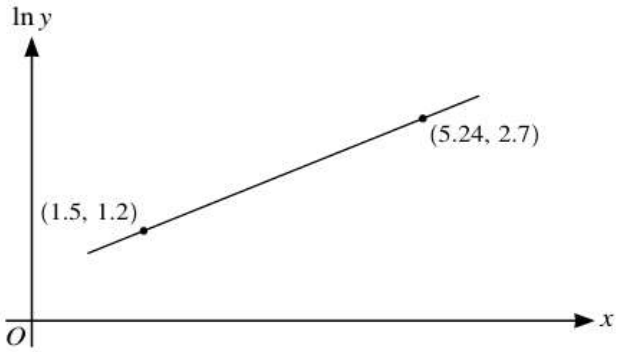
Mathematics 9709/32

Paper 3 Pure Mathematics 3 **May/June 2020**

Question No (2)

2



The variables x and y satisfy the equation $y^2 = Ae^{kx}$, where A and k are constants. The graph of $\ln y$ against x is a straight line passing through the points (1.5, 1.2) and (5.24, 2.7) as shown in the diagram.

Find the values of A and k correct to 2 decimal places.

Solution:

$y^2 = A e^{kx}$ → (1)

Taking ln on both sides

$\ln(y^2) = \ln(A e^{kx})$

ln properties

(i) $\ln x^n = n \ln x$

(ii) $\ln(xy) = \ln x + \ln y$

$2 \ln y = \ln A + \ln e^{kx}$

$2 \ln y = \ln A + kx \ln e$

$2 \ln y = \ln A + kx$ $\quad = \ln e = 1$

$\ln y = \frac{k}{2} x + \frac{1}{2} \ln A$ → (2)

Gradient formula for two points

$A(x_1, y_1)$, $B(x_2, y_2)$

gradient of $AB = \frac{y_2 - y_1}{x_2 - x_1}$

From the graph using (1.5, 1.2) and (5.24, 2.7), we have,

$$\text{gradient} = \frac{2.7-1.2}{5.24-1.5}$$

$$= \frac{1.5}{3.74}$$

$$= 0.401$$

standard linear equation ~~is~~ using gradient

$$y = mx + c$$

Here m is gradient and c is y-intercept.

comparing equation (2) with $y = mx + c$

$$\Rightarrow m = \frac{K}{2}$$

$$\Rightarrow 0.401 = \frac{K}{2}$$

$$K = 0.80$$

$m = 0.401$ from graph

As point $(1.5, 1.2)$ lie on equation (2)

$$1.2 = \frac{0.802}{2}(1.5) + \frac{1}{2}\ln A$$

$$1.2 = 0.6015 + \frac{1}{2}\ln A$$

$$\ln A = 1.197 \Rightarrow e^{\ln A} = e^{1.197}$$

$$A = 3.31$$

So value of $A = 3.31$ and $K = 0.802$