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Mathematics 9709/52

Paper 5 Probability & Statistics 1 **May/June 2023**

Question No (1)

1 The random variable X takes the values -2, 2 and 3. It is given that

$$P(X = x) = k(x^2 - 1),$$

where k is a constant.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions.

(b) Find $E(X)$ and $Var(X)$.

Solution:

(a)

$$P(X=x) = K(x^2-1) \rightarrow (1)$$

given values of x are

$-2, 2$ and 3

$$P(X=-2) = K((-2)^2-1)$$
$$= 3K$$

$$P(X=2) = K((2)^2-1)$$
$$= 3K$$

$$P(X=3) = K((3)^2-1)$$
$$P(X=3) = 8K$$

As the sum of probability is 1

$$\Rightarrow P(X=-2)+P(X=2)+P(X=3)=1$$

$$3K+3K+8K=1$$

$$\Rightarrow K = \frac{1}{14}$$

NOW

$$P(X=x) = K(x^2-1)$$

$$P(X=x) = \frac{1}{14}(x^2-1) \quad \because K = \frac{1}{14}$$

probability distribution table for X

x	-2	2	3
$P(x)$	$\frac{1}{14}((-2)^2-1) = \frac{3}{14}$	$\frac{1}{14}((2)^2-1) = 3/14$	$\frac{1}{14}((3)^2-1) = \frac{8}{14}$

(b)

$$\boxed{\begin{array}{c}\text{Knowledge sharing} \\ E(X) = \sum_{i=1}^{n} x_i P_i \\ \operatorname{var}(X) = \sum_{i=1}^{n} x_i^2 P_i - (E(X))^2\end{array}}$$

$$E(X) = \sum_{i=1}^{3} x_i P_i$$

$$E(X) = x_1 P_1 + x_2 P_2 + x_3 P_3$$

$$= (-2)(3/14) + (2)(3/14) + 3(8/14)$$

From (a) distribution table

$$E(X) = -6/14 + 6/14 + \frac{24}{14}$$

$$E(X) = \frac{12}{7}$$

Now $Var(X) = \sum_{i=1}^{3} x_i^2 p_i - (E(X))^2$

$= x_1^2 p_1 + x_2^2 p_2 + x_3^2 p_3 - (E(X))^2$

$= (-2)^2(3/14) + (2)^2(3/14) + (3)^2(8/14) - \left(\frac{12}{7}\right)^2$ $\quad \because E(X) = \frac{12}{7}$ and from table of part(a)

$= \frac{12}{14} + \frac{12}{14} + \frac{72}{14} - \frac{144}{49}$

$= \frac{84+84+504-288}{98}$

$= \frac{384}{98}$

$Var(X) = \frac{192}{49}$