Cambridge International AS & A Level

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Mathematics 9709/52

Paper 5 Probability & Statistics 1 October/November 2022

Question No (2)

2 The lengths of the rods produced by a company are normally distributed with mean 55.6 mm and standard deviation 1.2 mm.

(a) In a random sample of 400 of these rods, how many would you expect to have length less than 54.8 mm?

(b) Find the probability that a randomly chosen rod produced by this company has a length that is within half a standard deviation of the mean.

Solution:

(a) Mean, $\mu = 55.6$ m

standard deviation, $\delta = 1.2$ mm

Let the number of rods x be a function of normal distribution

P(Length is less than 54.8 m)

$P(X < 54.8)$ $\quad X \sim N(55.6, 1.2^2)$

$= P\left(\frac{X-\mu}{\delta} < \frac{54.8-55.6}{1.2}\right)$ standardizing x using $z = \frac{x-\mu}{\delta}$

$= P(Z < -0.667)$ $\quad Z \sim N(0,1)$

$= \phi(-0.667)$ $\quad \phi(Z<a) = \phi(a)$

$= 1 - \phi(0.667)$ $\quad \phi(-a) = 1-\phi(a)$

$= 1 - 0.7477$

$= 0.2523$

Expected number of rods $= 0.2523 \times 400$

$= 100.92$

≈ 101

(b) By given condition

$P(\mu - \frac{\delta}{2} < x < \mu + \frac{\delta}{2})$

$P\left(\frac{\mu - \frac{\delta}{2} - \mu}{\delta} < \frac{x-\mu}{\delta} < \frac{\mu + \frac{\delta}{2} - \mu}{\delta}\right)$

$= P(-0.5 < Z < 0.5)$

$= \phi(0.5) - \phi(-0.5)$

$= \phi(0.5) - [1 - \phi(0.5)]$

$= \phi(0.5) - 1 + \phi(0.5)$

$= 2\phi(0.5) - 1$

$= 2(0.6915) - 1$

$= 0.383$

$Z \sim N(0,1)$

$\therefore P(a < Z < b)$

$= \phi(b) - \phi(a)$