

Cambridge International AS & A Level

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Mathematics

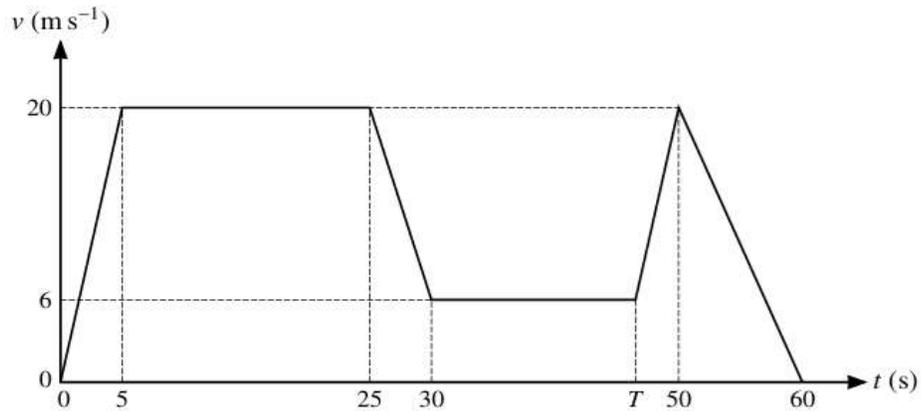
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Paper 4 Mechanics

October/November 2021

Question No (1)

1

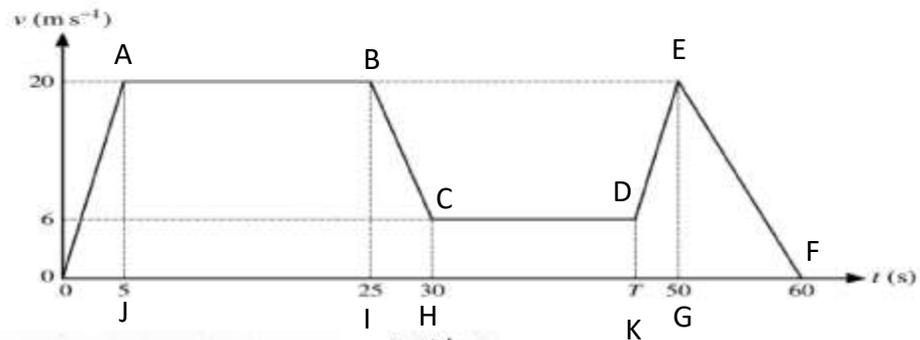


The diagram shows a velocity-time graph which models the motion of a car. The graph consists of six straight line segments. The car accelerates from rest to a speed of 20 m s^{-1} over a period of 5 s, and then travels at this speed for a further 20 s. The car then decelerates to a speed of 6 m s^{-1} over a period of 5 s. This speed is maintained for a further $(T - 30)$ s. The car then accelerates again to a speed of 20 m s^{-1} over a period of $(50 - T)$ s, before decelerating to rest over a period of 10 s.

- (a) Given that during the two stages of the motion when the car is accelerating, the accelerations are equal, find the value of T .
- (b) Find the total distance travelled by the car during the motion.

Solution:

② OA line shows the acceleration during the first stage



$$\text{Acceleration} = \frac{\text{velocity}}{\text{time}}$$

$$= \frac{OA}{OJ}$$

$$= \frac{20}{5}$$

$$a = 4 \text{ m s}^{-2}$$

DE line shows the acceleration during 2nd stage

$$\text{acceleration} = \frac{\text{velocity}}{\text{time}}$$

$$= \frac{DE}{KG}$$

$$a = \frac{14}{50-T}$$

As the acceleration of both stage equal

$$\Rightarrow 4 = \frac{14}{50-T}$$

$$50 - T = \frac{14}{4}$$

$$T = 50 - \frac{14}{4}$$

$$= \frac{200 - 14}{4}$$

$$T = 46.5 \text{ s}$$

(b)

Distance from 0 to 25 sec = Area of Trapezium OABI

$$= \frac{1}{2} (\text{sum of parallel sides}) \times \text{height}$$

$$= \frac{1}{2} (|OE| + |AB|) \times BI$$

$$= \frac{1}{2} (25 + 20) \times 20$$

$$= \frac{1}{2} (45) \times 20$$

$$d_1 = 450 \text{ m}$$

Distance from 25 to 30 = Area of Trapezium IBCH

$$= \frac{1}{2} (6 + 20) (5)$$

$$= \frac{1}{2} (26) (5)$$

$$d_2 = 65 \text{ m}$$

Distance from 30 to 46.5 sec = Area of Rectangle CDHF

$$= |CD| \times |CH|$$

$$= (46.5 - 30) \times 6$$

$$d_3 = 99 \text{ m}$$

Distance from 46.5 to 50 sec = Area of Trapezium KDEG

$$\begin{aligned}
 &= \frac{1}{2} (\text{sum of parallel sides}) \times \text{height} \\
 &= \frac{1}{2} (100 + 160) \times 10 \\
 &= \frac{1}{2} (6 + 20) (50 - 46.5) \\
 &= \frac{1}{2} (26) (3.5) \\
 d_4 &= 45.5 \text{ m}
 \end{aligned}$$

Distance from 50 to 60 sec = Area of triangle EGF

$$\begin{aligned}
 &= \frac{1}{2} \text{base} \times \text{height} \\
 &= \frac{1}{2} GF \times EG \\
 &= \frac{1}{2} (10) (20) \\
 d_5 &= 100 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{total distance} &= d_1 + d_2 + d_3 + d_4 + d_5 \\
 &= 450 + 65 + 99 + 45.5 + 100 \\
 &= 759.5 \text{ m}
 \end{aligned}$$