Cambridge International AS & A Level

Mathematics   9709

Paper 1 Pure Mathematics 1

Topic 8-Integration

Question No (22)

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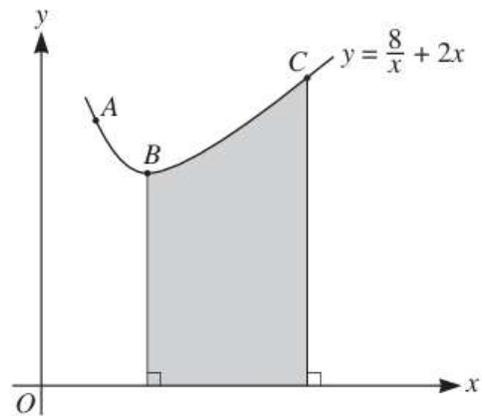
The diagram shows part of the curve $y = \dfrac{8}{x} + 2x$ and three points A, B and C on the curve with x-coordinates 1, 2 and 5 respectively.

(i) A point P moves along the curve in such a way that its x-coordinate increases at a constant rate of 0.04 units per second. Find the rate at which the y-coordinate of P is changing as P passes through A.

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis.

Solution

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Given curve

$$y = \frac{8}{x} + 2x$$

$$y = 8x^{-1} + 2x$$

Differentiate w.r.t x

$$\frac{dy}{dx} = 8\left[(-1)x^{-1-1}\right] + 2(1)$$

$$= -\frac{8}{x^2} + 2$$

at point A, $x = 1$

$$\frac{dy}{dx} = -\frac{8}{(1)^2} + 2$$

$$\frac{dy}{dx} = -8 + 2 = -6$$

As given in question

$$\frac{dx}{dt} = 0.04 \qquad \text{(rate of change of x)}$$

using the chain rule, we have

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$\frac{dy}{dt} = -6 \times 0.04$$

$$= -0.24$$

$\therefore$ rate of change of y- coordinate $= -0.24$ unit/sec

(ii)

volume of shaded region

$$v = \pi \int_a^b y^2 \, dx$$

$$= \pi \int_2^5 \left(\frac{8}{x} + 2x\right)^2 dx \qquad \begin{aligned} x_1 &= 2 \\ x_2 &= 5 \end{aligned}$$

$$= \pi \int_2^5 \left[\left(\frac{8}{x}\right)^2 + 2\left(\frac{8}{x}\right)(2x) + (2x)^2\right] dx$$

$$= \pi \int_2^5 \left(\frac{64}{x^2} + 32 + 4x^2\right) dx$$

$$= \pi \int_2^5 \left(64\,x^{-2} + 32 + 4x^2\right) dx$$

$$= \pi \left|64\, \frac{x^{-2+1}}{-2+1} + 32x + \frac{4x^3}{3}\right|_2^5$$

$$= \pi \left|64\, \frac{x^{-1}}{-1} + 32x + \frac{4}{3}x^3\right|_2^5$$

$$= \pi \left[\left(-\frac{64}{5} + 32(5) + \frac{4}{3}(5^3)\right) - \left(-\frac{64}{2} + 32(2) + \frac{4}{3}(2^3)\right)\right]$$

$$= \pi \left[-\frac{64}{5} + 160 + \frac{500}{3} + 32 - 64 - \frac{32}{3}\right]$$

$$= \frac{1356}{5}\pi$$

$$\approx 852 \text{ unit}^3$$