Cambridge International AS & A Level

Mathematics   9709

Paper 1 Pure Mathematics 1

Topic 8-Integration

Question No (21)

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The equation of a curve is $y = \dfrac{2}{\sqrt{(5x-6)}}$.

(i) Find the gradient of the curve at the point where $x = 2$.

(ii) Find $\displaystyle\int \dfrac{2}{\sqrt{(5x-6)}}\, dx$ and hence evaluate $\displaystyle\int_{2}^{3} \dfrac{2}{\sqrt{(5x-6)}}\, dx$.

<u>Solution</u>

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(i)   Equation of curve given

$$y = \frac{2}{\sqrt{5x-6}} \longrightarrow ①$$

$$y = 2(5x-6)^{-1/2}$$

Differentiate w.r.t x

$$\frac{dy}{dx} = 2\left[-\frac{1}{2}(5x-6)^{-1/2-1} \frac{d}{dx}(5x-6)\right]$$

$$= 2\left[-\frac{1}{2}(5x-6)^{-3/2}(5)\right]$$

$$\frac{dy}{dx} = -5(5x-6)^{-3/2}$$

when $x = 2$

$$\frac{dy}{dx} = -5\left(5(2)-6\right)^{-3/2}$$

$$= -5(10-6)^{-3/2}$$

$$= -5(4)^{-3/2}$$

$$= -5 \quad 2^{2 \times -3/2}$$

$$= -5 \times 2^{-3}$$

$$= -\frac{5}{2^3}$$

$$\frac{dy}{dx} = -\frac{5}{8}$$

(ii) $\displaystyle\int \frac{2}{\sqrt{5x-6}}\,dx$

$= 2\int (5x-6)^{-\frac{1}{2}}\,dx$

$= 2\ \dfrac{(5x-6)^{-\frac{1}{2}+1}}{\left(-\frac{1}{2}+1\right)(5)} + K$

$= 2\ \dfrac{(5x-6)^{\frac{1}{2}}}{\frac{1}{2}(5)}$

$= \dfrac{4}{5}\sqrt{5x-6} + K$

Now for $\displaystyle\int_{2}^{3}\frac{2}{\sqrt{5x-6}}\,dx$

$= \left|\dfrac{4}{5}\sqrt{5x-6}\right|_{2}^{3}$

$= \dfrac{4}{5}\sqrt{5(3)-6} - \dfrac{4}{5}\sqrt{5(2)-6}$

$= \dfrac{4}{5}\sqrt{9} - \dfrac{4}{5}\sqrt{4}$

$= \dfrac{4}{5}\times 3 - \dfrac{4}{5}\times 2$

$= \dfrac{12}{5} - \dfrac{8}{5}$

$= \dfrac{4}{5}$ Ans