Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 8-Integration

Question No (12)

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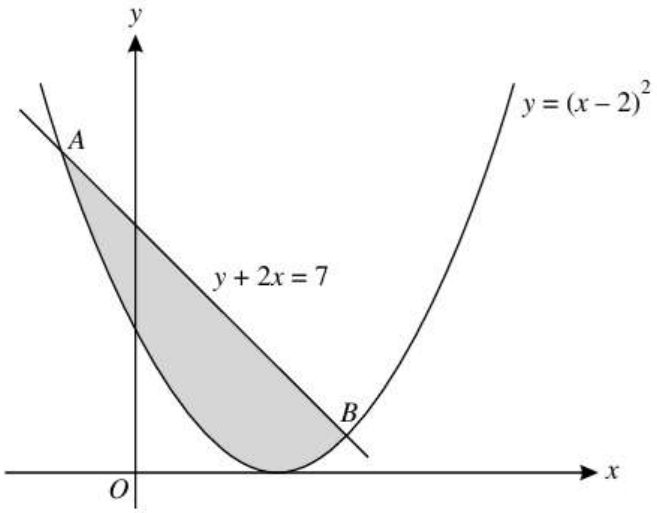

The diagram shows the curve $y = (x - 2)^2$ and the line $y + 2x = 7$, which intersect at points A and B. Find the area of the shaded region.

Solution

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Equation of curve

$$y = (x-2)^2 \rightarrow (1)$$

Equation of line

$$y + 2x = 7$$

$$y = 7 - 2x \rightarrow (2)$$

solving (1) & (2)

$$7 - 2x = (x-2)^2$$

$$7 - 2x = x^2 - 4x + 4$$

$$x^2 - 4x + 4 - 7 + 2x = 0$$

$$x^2 - 2x - 3 = 0$$

Factorize

$$x^2 - 3x + x - 3 = 0$$

$$x(x-3) + 1(x-3) = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3, x = -1$$

∴ lower limit $= -1$

upper limit $= 3$

Area of shaded region,

$$A = \int_{-1}^{3} [(\text{Equation of line}) - (\text{Equation of curve})]\,dx$$

$$= \int_{-1}^{3} [(7-2x) - (x-2)^2]\,dx$$

$$= \int_{-1}^{3} [7-2x-(x^2-4x+4)]\,dx$$

$$= \int_{-1}^{3} (7-2x-x^2+4x-4)\,dx$$

$$= \int_{-1}^{3} (3+2x-x^2)\,dx$$

$$= \left| 3x + \frac{2x^2}{2} - \frac{x^3}{3} \right|_{-1}^{3}$$

$$= \left| 3x + x^2 - \frac{x^3}{3} \right|_{-1}^{3}$$

$$= \left(3(3) + (3)^2 - \frac{(3)^3}{3}\right) - \left(3(-1) + (-1)^2 - \frac{(-1)^3}{3}\right)$$

$$= (9+9-9)\left(-3+1+\frac{1}{3}\right)$$

$$= 9 + 5/3 = \frac{32}{3} \text{ unit}^2$$