Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 8-Integration

Question No (11)

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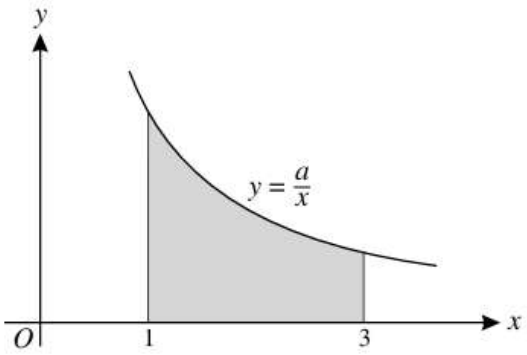

The diagram shows part of the curve $y = \frac{a}{x}$, where a is a positive constant. Given that the volume obtained when the shaded region is rotated through 360° about the x-axis is 24π, find the value of a.

Solution

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Given $y = \frac{a}{x}$

$$V = \pi \int_1^3 y^2 \, dx$$

$$= \pi \int_1^3 \left(\frac{a}{x}\right)^2 dx$$

$$= \pi \int_1^3 \frac{a^2}{x^2} \, dx$$

$$V = \pi a^2 \int_1^3 x^{-2} \, dx$$

$24\pi = \pi a^2 \int_1^3 x^{-2} \, dx$ $\quad V = 24\pi$ given

$$24 = a^2 \left| \frac{x^{-2+1}}{-2+1} \right|_1^3$$

$$24 = a^2 \left| \frac{x^{-1}}{-1} \right|_1^3$$

$$= a^2 \left| +\frac{1}{x} \right|_1^3$$

$$= a^2 \left| -\frac{1}{3} - \left(-\frac{1}{1}\right) \right|$$

$$24 = a^2 \left| -\frac{1}{3} + 1 \right| = a^2 \left(\frac{2}{3}\right)$$

$24 = a^2 \frac{2}{3}$

$a^2 = \frac{24 \times 3}{2}$

$a^2 = 36$

$a = \pm 6$

$a = 6$ ∵ a is +ve