Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 8-Integration

Question No (1)

<u>Question No (1)</u>

A curve is such that $\frac{dy}{dx} = \frac{4}{\sqrt{(6 - 2x)}}$, and $P\,(1,\ 8)$ is a point on the curve.

(i) The normal to the curve at the point P meets the coordinate axes at Q and at R. Find the coordinates of the mid-point of QR.

(ii) Find the equation of the curve.

<u>Solution</u>

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Given equation of curve

$$\frac{dy}{dx} = \frac{4}{\sqrt{6-2x}} \quad (1)$$

Gradient at $P(1,8)$ Equation (1) becomes

$$\frac{dy}{dx} = \frac{4}{\sqrt{6-2(1)}}$$

$$\frac{dy}{dx} = \frac{4}{\sqrt{4}} = \frac{4}{2} = 2$$

gradient of tangent × gradient of normal $= -1$

$\Rightarrow$ gradient of the normal $= -\frac{1}{2}$ (negative reciprocal of gradient of tangent)

Equation of normal passing through $P(1,8)$ is

$$y - y_1 = -\frac{1}{2}(x - x_1)$$

$$y - 8 = -\frac{1}{2}(x-1)$$

$$2(y-8) = -(x-1)$$

$$2y - 16 = -x + 1$$

$$x + 2y = 17$$

This normal meet the x-axis at Q.
Therefore Q is the x-intercept, put $y=0$
in equation of normal, we have

$$x+2(0)=17$$
$$x=17$$

$\therefore$ point $Q(17,0)$

Similarly the normal meets the y-axis
at point R.

$\therefore$ put $x=0$ in equation of normal
we have

$$0+2y=17$$
$$y=\frac{17}{2}$$

$\therefore$ point $R\left(0,\frac{17}{2}\right)$

Now.

mid-point of $QR=\left(\frac{x_1+x_2}{2},\ \frac{y_1+y_2}{2}\right)$

$$=\left(\frac{17+0}{2},\ \frac{0+\frac{17}{2}}{2}\right)$$

$$=\left(\frac{17}{2},\ \frac{17}{4}\right) \text{ Ans}$$

(ii) Now we shall find out the equation of
the curve.

As $$\frac{dy}{dx}=\frac{4}{\sqrt{6-2x}}$$

$dy = \frac{4}{\sqrt{6-2x}}\,dx$ (By seperating variables)

$dy = 4\,(6-2x)^{-1/2}\,dx$

integrating both sides.

$\int dy = \int 4\,(6-2x)^{-1/2}\,dx$

$y = 4\left[\frac{(6-2x)^{-1/2+1}}{(-1/2+1)(-2)}\right] + K$

$y = 4\left(\frac{(6-2x)^{1/2}}{(1/2)(-2)}\right) + K$

$y = -4\sqrt{6-2x} + K \rightarrow (2)$

$\int (2x)^5\,dx$

$= \frac{(2x)^{5+1}}{5+1(2)} + K$

As curve passes through point $P(1,8)$

Equation(2) becomes

$8 = -4\sqrt{6-2(1)} + K$

$8 = -4\sqrt{4} + K$

$8 = -8 + K$

$K = 16$

put $K = 16$ in (2)

$y = -4\sqrt{6-2x} + 16$

$y = 16 - 4\sqrt{6-2x}$