Cambridge International AS & A Level

Mathematics   9709

Paper 1 Pure Mathematics 1

Topic 7-Differentiation

Question No (32)

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A curve is such that $\dfrac{dy}{dx} = -x^2 + 5x - 4.$

(i) Find the *x*-coordinate of each of the stationary points of the curve.

(ii) Obtain an expression for $\dfrac{d^2y}{dx^2}$ and hence or otherwise find the nature of each of the stationary points.

(iii) Given that the curve passes through the point (6, 2), find the equation of the curve.

Solution

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Given equation of curve

$$\frac{dy}{dx} = -x^2 + 5x - 4 \longrightarrow ①$$

(i)

For the stationary point, $\frac{dy}{dx} = 0$

$$\Rightarrow -x^2 + 5x - 4 = 0$$

$$-(x^2 - 5x + 4) = 0$$

$$x^2 - 5x + 4 = 0$$

Factorize

$$x^2 - 4x - x + 4 = 0$$

$$x(x-4) - 1(x-4) = 0$$

$$(x-4)(x-1) = 0$$

$$x - 4 = 0, \quad x - 1 = 0$$

$$x = 4, \quad x = 1$$

$$\therefore x = 4, x = 1$$

(ii)

As $\frac{dy}{dx} = -x^2 + 5x - 4$ from (1)

differentiate w.r.t x

$$\frac{d^2y}{dx^2} = -2x + 5$$

at $x=1$

$$\frac{d^2y}{dx^2} = -2(1)+5$$

$$= 3 > 0$$

∴ stationary point is minimum at $x=1$

Now at $x=4$

$$\frac{d^2y}{dx^2} = -2(4)+5$$

$$= -8+5$$

$$= -3 < 0$$

∴ stationary point is maximum

at $x=4$.

(iii)

As $\frac{dy}{dx} = -x^2+5x-4$ From (i)

$\int dy = \int (-x^2+5x-4)dx$ seperate variables

$$y = -\frac{x^3}{3} + \frac{5x^2}{2} - 4x + K$$

Since $(6,2)$ lies on curve

$$2 = -\frac{(6)^3}{3} + \frac{5}{2}(6)^2 - 4(6) + K$$

$$2 = -72 + 90 - 24 + K$$

$$\Rightarrow 2 = -6 + k$$
$$k = 8$$

$\therefore$ Equation of curve is

$$y = -\frac{x^3}{3} + \frac{5}{2}x^2 - 4x + 8$$