Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 7-Differentiation

Question No (31)

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The equation of a curve is $y = 8\sqrt{x} - 2x$.

(i) Find the coordinates of the stationary point of the curve.

(ii) Find an expression for $\frac{d^2y}{dx^2}$ and hence, or otherwise, determine the nature of the stationary point.

(iii) Find the values of x at which the line $y = 6$ meets the curve.

(iv) State the set of values of k for which the line $y = k$ does not meet the curve.

Solution

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(c) Equation of curve

$y = 8\sqrt{x} - 2x \rightarrow (1)$

$y = 8(x)^{1/2} - 2x$

Differentiate w.r.t x

$\frac{dy}{dx} = 8\left[\frac{1}{2}x^{1/2-1}\right] - 2(1)$

$= 4x^{-1/2} - 2$

$\frac{dy}{dx} = \frac{4}{\sqrt{x}} - 2$

For stationary point $\frac{dy}{dx} = 0$

$\frac{4}{\sqrt{x}} - 2 = 0$

$\frac{4}{\sqrt{x}} = 2$

$\sqrt{x} = 2$

squaring

$x = 4$

put $x = 4$ in (1)

$y = 8(\sqrt{4}) - 2(4)$

$= 8(2) - 8$

$= 16 - 8$

$y = 8$

∴ coordinates of the stationary points are
(4,8)

(ii)

As $\frac{dy}{dx} = \frac{4}{\sqrt{x}} - 2$ From (i)

$= \frac{4}{(x)^{1/2}} - 2$

$\frac{dy}{dx} = 4x^{-1/2} - 2$

Differentiate w.r.t x

$\frac{d^2y}{dx^2} = 4\left[-\frac{1}{2}x^{-1/2-1}\right] - 0$

$= -2x^{-3/2}$

At (4, 8)

$\frac{d^2y}{dx^2} = -2\,(4)^{-3/2}$

$= -2\,(2^2)^{-3/2}$

$= -2\,2^{-3}$

$\frac{d^2y}{dx^2} = -\frac{2}{2^3} = -\frac{2}{8} = -\frac{1}{4} < 0$

∴ stationary point (4,8) is a maximum point.

(iii) put $y=6$ in equation (1) of curve

$6 = 8\sqrt{x} - 2x$

$8\sqrt{x} - 2x - 6 = 0 \rightarrow (2)$

let $\sqrt{x} = t$

$x = t^2$

Equation (2) becomes

$8t - 2(t)^2 - 6 = 0$

$8t - 2t^2 - 6 = 0$

$-(2t^2 - 8t + 6) = 0$

$2t^2 - 8t + 6 = 0 \Rightarrow 2(t^2 - 4t + 3) = 0$

Factorize $\qquad t^2 - 4t + 3 = 0$

$t^2 - 3t - t + 3 = 0$

$t(t-3) - 1(t-3) = 0$

$(t-3)(t-1) = 0$

$t - 3 = 0, \quad t - 1 = 0$

$t = 3, \quad t = 1$

$\sqrt{x} = 3, \quad \sqrt{x} = 1 \qquad \because t = \sqrt{x}$

$x = 9, \quad x = 1$

both values of x satisfy the equation of curve,

$\therefore x = 1, x = 9$

(iv) substitute $y = k$ in equation (1)

$8\sqrt{x} - 2x = k$

$2x - 8\sqrt{x} + k = 0$

since line does not meet the curve

discriminant, $b^2 - 4ac < 0$

$(-8)^2 - 4(2)(k) < 0$

$64 - 8k < 0$

$-8k < -64$

$8k > 64$

$k > 8$