Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 7-Differentiation

Question No (22)

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Question No (22)

The equation of a curve is $y = x^3 + ax^2 + bx$, where a and b are constants.

(i) In the case where the curve has no stationary point, show that $a^2 < 3b$.

(ii) In the case where $a = -6$ and $b = 9$, find the set of values of x for which y is a decreasing function of x.

Solution

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(ii) $y = x^3 + ax^2 + bx \rightarrow (1)$

Differentiate w.r.t x

$$\frac{dy}{dx} = 3x^2 + 2ax + b$$

For stationary point, $\frac{dy}{dx} = 0$

$\Rightarrow 3x^2 + 2ax + b = 0$

curve has no stationary point when

discriminant $= b^2 - 4ac < 0$

$\Rightarrow (2a)^2 - 4(3)(b) < 0$

$4a^2 - 12b < 0$

$4a^2 < 12b$

$a^2 < 3b$ proved.

(iii) As Given

$a = -6$ and $b = 9$

put in equation (1)

$y = x^3 - 6x^2 + 9x$

$\frac{dy}{dx} = 3x^2 - 6(2x) + 9$

$\frac{dy}{dx} = 3x^2 - 12x + 9$

y is decreasing function if

$$\frac{dy}{dx} < 0$$

$$\Rightarrow 3x^2 - 12x + 9 < 0$$

$$3(x^2 - 4x + 3) < 0$$

$$x^2 - 4x + 3 < 0$$

By factorization

$$x^2 - 3x - x + 3 < 0$$

$$x(x-3) - 1(x-3) < 0$$

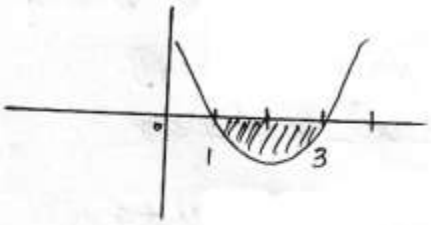
$$(x-3)(x-1) < 0$$

critical values

$x - 3 = 0, \; x - 1 = 0$

$x = 3, \; x = 1$

As $(x-3)(x-1) < 0$



$\therefore \quad 1 < x < 3$ curve below x-axis