Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 7-Differentiation

Question No (21)

Question No (21)

A curve has equation $y = \frac{12}{3-2x}$.

(i) Find $\frac{dy}{dx}$.

A point moves along this curve. As the point passes through A, the x-coordinate is increasing at a rate of 0.15 units per second and the y-coordinate is increasing at a rate of 0.4 units per second.

(ii) Find the possible x-coordinates of A.

Solution

On Next page

(i) $y = \frac{12}{3-2x} \rightarrow (1)$

$y = 12(3-2x)^{-1}$

Differentiate w.r.t x

$\frac{dy}{dx} = 12\left[(-1)(3-2x)^{-1-1}\frac{d}{dx}(3-2x)\right]$

$= 12\left[(-1)(3-2x)^{-2}(0-2)\right]$

$= 12\left[2(3-2x)^{-2}\right]$

$= 24(3-2x)^{-2}$

$\frac{dy}{dx} = \frac{24}{(3-2x)^2}$

(ii) As Given at point A, $\frac{dx}{dt} = 0.15$ and

$\frac{dy}{dt} = 0.4$

By chain rule

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{dy}{dx} = 0.4 \times \frac{1}{0.15} = \frac{0.4}{0.15}$

$\frac{dy}{dx} = \frac{0.4}{0.15}$

$\Rightarrow \quad \frac{24}{(3-2x)^2} = \frac{0.4}{0.15}$ $\qquad \therefore \frac{dy}{dx} = \frac{24}{(3-2x)^2}$ from (1)

$(3-2x)^2 = \frac{24 \times 0.15}{0.4}$

$(3-2x)^2 = 9$

$3-2x = \pm 3$

$3-2x = 3$	$3-2x = -3$
$-2x = 3-3$	$-2x = -3-3$
$-2x = 0$	$-2x = -6$
$x = 0$	$x = 3$

$\therefore$ possible x-coordinates of point A are 0 or 3.