

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 7-Differentiation

Question No (11)

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Question No (11)

The equation of a curve is $y = \frac{1}{6}(2x - 3)^3 - 4x$.

(i) Find $\frac{dy}{dx}$.

(ii) Find the equation of the tangent to the curve at the point where the curve intersects the y -axis.

(iii) Find the set of values of x for which $\frac{1}{6}(2x - 3)^3 - 4x$ is an increasing function of x .

Solution

On Next page

Equation of curve

$$y = \frac{1}{6}(2x-3)^3 - 4x \rightarrow (1)$$

(i) Differentiate (1) w.r.t x

$$\frac{dy}{dx} = \frac{1}{6} [3(2x-3)^{3-1} \frac{d}{dx}(2x-3)] - 4(1)$$

$$= \frac{1}{6} [3(2x-3)^2 (2)] - 4$$

$$\frac{dy}{dx} = (2x-3)^2 - 4$$

(ii) curve $y = \frac{1}{6}(2x-3)^3 - 4x \rightarrow (2)$

As the curve intersects the y -axis,
so put $x = 0$ in (2)

$$y = \frac{1}{6}(2(0)-3)^3 - 4(0)$$

$$= -\frac{27}{6}$$

$$y = -\frac{9}{2}$$

\therefore The curve meets the y -axis at point
 $(0, -\frac{9}{2})$.

Now at $(0, -\frac{9}{2})$ gradient of tangent is

$$\frac{dy}{dx} = (2x-3)^2 - 4 \quad \text{from part (i)}$$

$$\text{at } (0, -9/2)$$

$$\frac{dy}{dx} = (2(0)-3)^2 - 4$$

$$= 9 - 4$$

$$\frac{dy}{dx} = 5$$

∴ Equation of tangent

$$y - y_1 = \frac{dy}{dx} (x - x_1)$$

$$y - (-9/2) = 5(x - 0)$$

$$y + 9/2 = 5x$$

$$2y + 9 = 5x(2)$$

$$2y + 9 = 10x$$

$$2y - 10x + 9 = 0$$

(ii)

From part (i)

$$\frac{dy}{dx} = (2x-3)^2 - 4$$

For increasing function

$$\frac{dy}{dx} > 0$$

$$\therefore \frac{dy}{dx} = 5$$

$$P(x, y) = (0, -9/2)$$

$$\Rightarrow (2x-3)^2 - 4 > 0$$

$$(2x-3)^2 - (2)^2 > 0$$

$$(2x-3+2)(2x-3-2) > 0$$

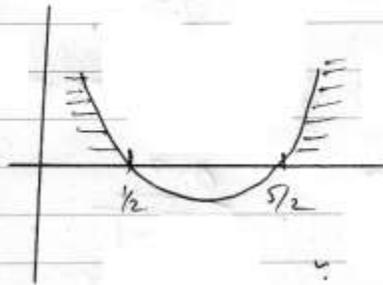
$$(2x-1)(2x-5) > 0$$

$$\checkmark a^2 - b^2 = (a+b)(a-b)$$

critical values

$$2x-1=0, \quad 2x-5=0$$

$$x = \frac{1}{2}, \quad x = \frac{5}{2}$$



$$\therefore x < \frac{1}{2}, x > \frac{5}{2}$$

$$\checkmark (2x-1)(2x-5) > 0$$

$$\Rightarrow y > 0$$

So we shall

take above

x-axis graph

