Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 7-Differentiation

Question No (2)

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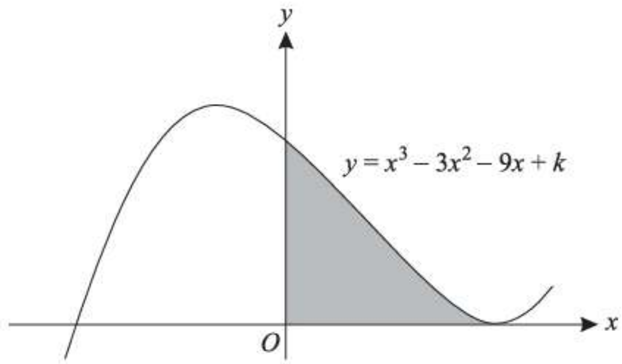

The diagram shows the curve $y = x^3 - 3x^2 - 9x + k$, where k is a constant. The curve has a minimum point on the x-axis.

(i) Find the value of k.

(ii) Find the coordinates of the maximum point of the curve.

(iii) State the set of values of x for which $x^3 - 3x^2 - 9x + k$ is a decreasing function of x.

(iv) Find the area of the shaded region.

Solution

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(i) Given equation of curve

$$y = x^3 - 3x^2 - 9x + K \longrightarrow (1)$$

Differentiate (1) w.r.t x

$$\frac{dy}{dx} = 3x^{3-1} - 3(2x^{2-1}) - 9(1) + 0$$

$$\frac{dy}{dx} = 3x^2 - 6x - 9$$

~~As~~ For max or min put $\frac{dy}{dx} = 0$

$$\Rightarrow 3x^2 - 6x - 9 = 0$$

$$3(x^2 - 2x - 3) = 0$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

By factorization

$$x^2 - 3x + x - 3 = 0$$

$$x(x-3) + 1(x-3) = 0$$

$$(x-3)(x+1) = 0$$

$$x - 3 = 0, \quad x + 1 = 0$$

$$x = 3, \quad x = -1$$

By observing the diagram and values of x, we see that curve has minimum point at $x = 3$, $\therefore$ minimum point of the curve is $(3, 0)$ as it lies on the x-axis.

As (3,0) lie on curve, so equation ① becomes

$0 = (3)^3 - 3(3)^2 - 9(3) + K$

$0 = 27 - 27 - 27 + K$

$\Rightarrow K = 27$ Ans

(ii) Now we shall find out the coordinates of the max point.
From part (i) and diagram of the curve, we see that curve is maximum at $x = -1$

put $x = -1$ in Equation (1)

$y = x^3 - 3x^2 - 9x + K$

$y = (-1)^3 - 3(-1)^2 - 9(-1) + 27$ ∵ $K = 27$ part (i)

$= -1 - 3 + 9 + 27$

$y = 32$

∴ coordinates of maximum point are $(-1, 32)$.

(iii) As $y = x^3 - 3x^2 - 9x + K$ given

Differentiate w.r.t x

$\frac{dy}{dx} = 3x^2 - 3(2x) - 9 + 0$

$\frac{dy}{dx} = 3x^2 - 6x - 9$

For decreasing function

$\frac{dy}{dx} < 0$

$\Rightarrow 3x^2 - 6x - 9 < 0$ $\qquad \because \frac{dy}{dx} = 3x^2 - 6x - 9$

$3(x^2 - 2x - 3) < 0$

$\Rightarrow x^2 - 2x - 3 < 0$

$x^2 - 3x + x - 3 < 0$ $\qquad$ By factorization

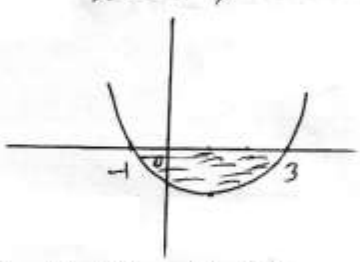
$x(x-3) + 1(x-3) < 0$

$(x-3)(x+1) < 0$

critical values

$x - 3 = 0, \quad x + 1 = 0$

$x = 3, \quad x = -1$



As $x^2 - 2x - 3 = 0$

$a = 1 > 0$

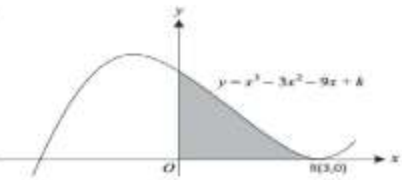
so cup is up and it will have min value.

As $(x-3)(x+1) < 0$

we shall consider the values below the horizontal axis

$\Rightarrow$ required set of values of x are $-1 < x < 3$

(iv) Now we shall find out the area of shaded region



$$\text{Area of Shaded region} = \int_0^3 y\,dx$$

$$= \int_0^3 (x^3 - 3x^2 - 9x + 27)\,dx$$

$$= \left| \frac{x^4}{4} - \frac{3x^3}{3} - \frac{9x^2}{2} + 27x \right|_0^3$$

Formula

$$\int x\,dx = \frac{x^{1+1}}{2}$$

$$= \left[\frac{(3)^4}{4} - \frac{3(3)^3}{3} - \frac{9}{2}(3)^2 + 27(3)\right] - \left[\frac{(0)^4}{4} - \frac{3(0)^3}{3} - \frac{9(0)^2}{2} + 27(0)\right]$$

$$= \frac{81}{4} - 27 - \frac{81}{2} + 81$$

$$= \frac{81 - 108 - 162 + 324}{4}$$

$$= \frac{135}{4} \text{ square units}$$