Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 6-Series

Question No (22)

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(a) The first two terms of an arithmetic progression are 16 and 24. Find the least number of terms of the progression which must be taken for their sum to exceed 20 000.

(b) A geometric progression has a first term of 6 and a sum to infinity of 18. A new geometric progression is formed by squaring each of the terms of the original progression. Find the sum to infinity of the new progression.

Solution

On Next page

(a) By Question Statement

First term, $a = 16$

second term $= a+d = 24$

common difference, $d =$ second term - First term

$= 24 - 16$

$d = 8$

By question condition

$S_n > 20000$ (Sum to exceed 20000)

$\Rightarrow \frac{n}{2}[2a + (n-1)d] > 20,000.$ $\because S_n = \frac{n}{2}[2a+(n-1)d]$ sum of nth term in A.P

$\frac{n}{2}[2\times16 + (n-1)8] > 20,000$

$n[32 + 8n - 8] > 2\times 20,000$

$n[32 + 8n - 8] > 40,000$

$32n + 8n^2 - 8n > 40,000$

$8n^2 + 24n - 40,000 > 0$

$8(n^2 + 3n - 5000) > 0$

$\Rightarrow n^2 + 3n - 5000 > 0$

using quadratic formula

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-5000)}}{2 \times 1}$$

$$= \frac{-3 \pm \sqrt{9 + 20000}}{2}$$

$$= \frac{-3 \pm \sqrt{20009}}{2}$$

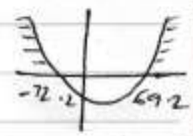
$$= \frac{-3 \pm 141.45}{2}$$

$$= \left(\frac{-3 + 141.45}{2}, \frac{-3 - 141.45}{2} \right)$$

$$n = (69.2, -72.2)$$

so quadratic inequality becomes

$$(n - 69.2)(n + 72.2) > 0$$



$\Rightarrow$ $n > 69.2$, $n < -72.2$ (ignore as n is not negative)

∴ The least number of terms required = 70

(b) Data given

First term, $a = 6$

$S_\infty = 18$

$\Rightarrow \frac{a}{1-r} = 18$ $\quad \because S_\infty = \frac{a}{1-r}$

$\frac{6}{1-r} = 18$

$\frac{6}{18} = 1-r$

$\frac{1}{3} = 1-r$

$r = 1 - \frac{1}{3}$

$r = \frac{3-1}{3} = 2/3$

∴ Geometric progression is

a, ar, ar^2

$\Rightarrow 6, 6(2/3), 6(2/3)^2, \ldots$

By condition new G.P will be

$(6)^2, (6(2/3))^2, (6(2/3)^2)^2, \ldots$

$36, 36 \times \frac{4}{9}, 36 \times \frac{16}{81}, \ldots$

$36, 16, \ldots$

$\therefore$ For this progression

$$a = 36, \quad r = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Sum to infinity

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{36}{1-4/9}$$

$$= \frac{36}{9-4/9}$$

$$= 36 \times \frac{9}{5}$$

$$= \frac{324}{5}$$

$$S_\infty = 64.8$$