Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 6-Series

Question No (21)

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(a) A cyclist completes a long-distance charity event across Africa. The total distance is 3050 km. He starts the event on May 1st and cycles 200 km on that day. On each subsequent day he reduces the distance cycled by 5 km.

(i) How far will he travel on May 15th?

(ii) On what date will he finish the event?

(b) A geometric progression is such that the third term is 8 times the sixth term, and the sum of the first six terms is $31\frac{1}{2}$. Find

(i) the first term of the progression,

(ii) the sum to infinity of the progression.

Solution

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(a)(i) By question statement

Distance covered on 1st May = 200 km

Distance covered on 2nd May = 195 ∴ in subsequent day 5 km reduces.

Distance covered on 3rd May = 190 km

∴ The distance can be represented by an arithmetic progression as on each subsequent day the distance reduces by 5 km.

∴ A.P is

200, 195, 190, ----

First term, $a = 200$,

common difference, d = Second term − First term

$= 195 - 200$

$= -5$

$n = 15$ (we need to calculate on 15th May) using nth term formula for A.P

$T_n = a + (n-1)d$

at $n = 15$

$T_{15} = 200 + (15-1)(-5)$ ∴ $a = 200$, $d = -5$

$= 200 + 14(-5)$

$= 200 - 70 = 130$

∴ distance travelled on 15th day = 130 km

(ii) As given in statement

$S_n = 3052$ km

using sum of nth term formula for A.P, we have

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$3052 = \frac{n}{2}[2(200) + (n-1)(-5)]$$

$$2 \times 3052 = n[400 - 5n + 5]$$

$$6100 = n[405 - 5n]$$

$$6100 = 405n - 5n^2$$

$$5n^2 - 405n + 6100 = 0$$

$$5(n^2 - 81n + 1220) = 0$$

$$\Rightarrow n^2 - 81n + 1220 = 0$$

using the quadratic formula

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-81) \pm \sqrt{(-81)^2 - 4(1)(1220)}}{2 \times 1}$$

$$= \frac{81 \pm \sqrt{6561 - 4880}}{2}$$

$$n = \frac{81 \pm \sqrt{16\,81}}{2}$$

$$= \frac{81 \pm 41}{2}$$

$$= \left(\frac{81+41}{2}, \frac{81-41}{2}\right)$$

$$= \left(61, \frac{40}{2}\right)$$

$$n = (61, 20)$$

$n = 61$ (ignore as 61 is not date)

$\therefore$ $n = 20$

cyclist will finish the event on 20th May.

(b)(i) By question statement

$T_3 = 8\,T_6$ (Third term is 8 times the sixth term)

$\therefore$ in G.P $a_n = ar^{n-1}$

$a_3 = ar^2$

$a_6 = ar^5$

$ar^2 = 8ar^5$

$\frac{1}{8} = r^3$

$(r)^3 = \left(\frac{1}{2}\right)^3$

$\Rightarrow r = 1/2$

sum of first six terms is $31\frac{1}{2}$

$\Rightarrow S_6 = 31\frac{1}{2}$

By using sum of nth formula for G.P we have

$$S_n = \frac{a(1-r^n)}{1-r}$$

at $n=6$

$$S_6 = \frac{a(1-r^6)}{1-r}$$

$$31\frac{1}{2} = \frac{a(1-r^6)}{1-r}$$

$$31.5 = \frac{a(1-(1/2)^6)}{1-1/2}$$

$$31.5 = \frac{a(1-\frac{1}{64})}{2-1/}$$

$$= a\left(\frac{64-1}{64}\right) \times \frac{2}{1}$$

$$31.5 = a\left(\frac{63}{64}\right) \times \frac{2}{1}$$

$$a = 31.5 \times \frac{64}{63 \times 2}$$

$$a = 16$$

(ii) For sum to infinity

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{16}{1-1/2} \qquad \therefore a = 16, \; r = 1/2$$

$$= \frac{16}{2-1/2}$$

$$= 16 \times \frac{2}{1}$$

$$S_\infty = 32$$