

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 6-Series

Question No (12)

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Question No (12)

- (a) An arithmetic progression contains 25 terms and the first term is -15 . The sum of all the terms in the progression is 525. Calculate
- (i) the common difference of the progression,
 - (ii) the last term in the progression,
 - (iii) the sum of all the positive terms in the progression.
- (b) A college agrees a sponsorship deal in which grants will be received each year for sports equipment. This grant will be \$4000 in 2012 and will increase by 5% each year. Calculate
- (i) the value of the grant in 2022,
 - (ii) the total amount the college will receive in the years 2012 to 2022 inclusive.

Solution

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②(i)

By given data
contains 25 terms $\Rightarrow n=25$

and first term is

$$a = -15$$

$$\text{and } S_{25} = 525$$

Formula for the sum of n th
terms for A.P is

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\Rightarrow S_{25} = \frac{25}{2} [2a + (25-1)d]$$

$$525 = \frac{25}{2} [2a - 15 + 24d] \quad \text{& } S_{25} = 525$$

$$a = -15$$

$$\frac{2 \times 525}{25} = [-30 + 24d]$$

$$42 = -30 + 24d$$

$$42 + 30 = 24d$$

$$\frac{42 + 30}{24} = d$$

$$\Rightarrow d = 3$$

common difference is 3.

(ii) Now we find last term in the A.P.
AS

$$T_n = a + (n-1)d$$

$$T_{25} = a + (25-1)d$$

$$= -15 + (24)(3) \quad \because d=3$$

$$= -15 + 72$$

$$T_{25} = 57$$

$\therefore$ Last term is 57.

(iii) AS $a = -15$, $d = 3$, $a_n = 57$ from (i)

$$a_n = a + (n-1)d$$

$$57 = -15 + (n-1)3$$

$$57 + 15 = (n-1)3$$

$$72 = (n-1)3$$

$$n-1 = \frac{72}{3}$$

$$n-1 = 24$$

$$n = 24 + 1$$

$$n = 25$$

AS in A.P. term total terms are

-15, -12, -9, -6, -3, 0, 3, 6, 9, 12, ..., 57

$\therefore$ number of terms are 20

$\because$ (-15, -12, -9, -6, -3, 0, 3, 6, 9, 12, ..., 57) are -ve

Now sum of all positive terms are

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{20} = \frac{20}{2} [2(0) + (20-1)3]$$

$$= 10 [0 + (19)(3)]$$

$$= 10 (57)$$

$$S_{20} = 570$$

$\therefore$ Sum of the terms is 570.

(b)(i) By Question statement

Let the grant in 2012 be $P = \$4000$

As the grant increase 5%, each year

$$\therefore \text{grant in 2013} = P + 5\% \text{ of } P$$

$$= P + 0.05 P = P(1 + 0.05)$$

$$= (1.05) P$$

$$\therefore \text{grant in 2014} = (1.05) P + 5\% \text{ of } (1.05) P$$

$$= (1.05) P + 0.05 (1.05) P$$

$$= (1 + 0.05) (1.05) P$$

$$= (1.05)^2 P$$

By following this movement, the sequence of grant from 2012 to 2022 is

$$P, (1.05)P, (1.05)^2P, \dots, (1.05)^{10}P$$

$$\therefore \text{value of grant in 2022} = (1.05)^{10}P$$

$$= (1.05)^{10} (4000)$$

$$= \$6515$$

$$= \$6520$$

(ii) As from 2012 $\rightarrow$ 2022 there is 11 years,

so total amount in 11 years is given by

$$P + (1.05)P + (1.05)^2P + \dots + (1.05)^{10}P$$

This is a G.P with

$$A = P = \$4000, r = 1.05 \text{ and } n = 11$$

As $r = 1.05 > 1$, so we shall use

Sum of G.P formula

$$S_n = A \left(\frac{r^n - 1}{r - 1} \right)$$

$$S_{11} = 4000 \left(\frac{(1.05)^{11} - 1}{1.05 - 1} \right)$$

$$= \frac{4000(1.7103 - 1)}{0.05}$$

$$= \frac{4000(0.7103)}{0.05}$$

$$= 56827.2$$

$\therefore$ total amount the college will receive = \$ 56827.2

$$\approx 56800$$