Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 6-Series

Question No (2)

Question No (2)

(a) Find the sum of all the integers between 100 and 400 that are divisible by 7.

(b) The first three terms in a geometric progression are 144, x and 64 respectively, where x is positive. Find

(i) the value of x,

(ii) the sum to infinity of the progression.

Solution

On Next page

We need to find out the integers divisible by 7 between 100 and 400.

105 is the first integer divisible by 7.

112 is the second integer divisible by 7.

So 105, 112, 119, ... 399 are the integers divisible by 7.

So

$105 + 112 + 119 + \ldots\ldots + 399$ is a arithmetic series in which

$a = 105$, d = common difference

= Second term − First term

$= 112 - 105$

$d = 7$

$n = ?$

nth term formula in Arithmetic series $T_n = a + (n-1)d$

$\therefore$ using $T_n = a + (n-1)d$

$399 = 105 + (n-1)7$

$$399 - 105 = (n-1)7$$

$$294 = (n-1)7$$

$$\frac{294}{7} = (n-1)$$

$$42 = n-1$$

$$n = 42+1$$

$$n = 43$$

Formula for arithmetic sum when last term is given.

$$S_n = \frac{n}{2}(a+l)$$

using formula

$$S_n = \frac{n}{2}(a+l)$$

$$S_{43} = \frac{43}{2}(105+399)$$

$$= \frac{43}{2}(504)$$

$$S_{43} = 10836$$

(b)(i) As given in question statement, first three terms in a G.P are

144, x, 64

As common ratio, $r = \frac{\text{second term}}{\text{First term}}$

is same for all consective terms, so this common ratio is equal.

$\Rightarrow$ common ratio, $r = \frac{x}{144}$ between 2nd and First term

and

common ratio, $r = \frac{64}{x}$ between Third and second term

$\Rightarrow \frac{x}{144} = \frac{64}{x}$ $\because$ common ratios are equal between consective terms

$\Rightarrow$

$x^2 = 144 \times 64$

$x^2 = 9216$

$x = \pm 96$

As x is +ve, so

$x = 96$

(ii) For the sum to infinity of the progression

common ratio, $r = \frac{\text{second term}}{\text{First term}}$

$= \frac{96}{144}$

$= \frac{2}{3}$

As $a = 144$

Formula for sum to infinity for Geometric progression

$S_\infty = \frac{a}{1-r}$

$\Rightarrow S_\infty = \frac{a}{1-r}$

$= \frac{144}{1-\frac{2}{3}}$

$= \frac{144}{1/3}$

$= 144 \times \frac{3}{1}$

$S_\infty = 432$