Cambridge International AS & A Level

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Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 6-Binomial Expansion

Question No (12)

**Question No (12)**

The coefficient of $x^3$ in the expansion of $(a + x)^5 + (2 - x)^6$ is 90. Find the value of the positive constant $a$.

**Solution**

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## Binomial expansion

$$(a+b)^n = {}^nC_0\, a^n b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 + \cdots + {}^nC_n\, a^0 b^n$$

$(a+x)^5 + (2-x)^6$

using the binomial expansion

$= \left({}^5C_0(a^5)(x^0) + {}^5C_1\, a^4 x^1 + {}^5C_2\, a^3 x^2 + {}^5C_3\, a^2 x^3 + \cdots\right) +$

$\left({}^6C_0(2^6)(x^0) + {}^6C_1(2^5)(-x)^1 + {}^6C_2\, 2^4 (x)^2 + {}^6C_3\, 2^3 (-x)^3 + \cdots\right)$

$= (a^5 + 5a^4x + 10a^3x^2 + 10a^2x^3 + \cdots)$

$+ (64 - 192x + 240x^2 - 160x^3 + \cdots)$

terms containing $x^3$, only

$10a^2x^3 - 160x^3$

$(10a^2 - 160)x^3$

As coefficient of $x^3$ is 90

$\Rightarrow 10a^2 - 160 = 90$

$10a^2 = 90 + 160 = 250$

$10a^2 = 250 \Rightarrow a^2 = 25$

$a = \pm 5$

$\therefore a = 5$