

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 6-Binomial Expansion

Question No (21)

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Question No (21)

- (i) Find the coefficient of x in the expansion of $\left(2x - \frac{1}{x}\right)^5$.
- (ii) Hence find the coefficient of x in the expansion of $(1 + 3x^2)\left(2x - \frac{1}{x}\right)^5$.

Solution

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(i) $\left(2x - \frac{1}{x}\right)^5$

Binomial Expansion

$$(a+b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_n a^0 b^n$$

General Term Formula

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

General Term

$$T_{r+1} = {}^5C_r (2x)^{5-r} \left(-\frac{1}{x}\right)^r$$

$$= {}^5C_r (2)^{5-r} x^{5-r} (-1)^r \left(\frac{1}{x}\right)^r$$

$$= {}^5C_r (2)^{5-r} x^{5-r} (-1)^r \left(\frac{1}{x}\right)^r$$

$$= {}^5C_r (2)^{5-r} x^{5-r} (-1)^r \frac{1}{x^r}$$

$$= {}^5C_r (2)^{5-r} (-1)^r x^{5-r-r}$$

$$= {}^5C_r (2)^{5-r} (-1)^r x^{5-2r} \longrightarrow \textcircled{1}$$

For the coefficient of x

$$x^{5-2r} = x^1$$

$$\Rightarrow 5-2r=1$$

$$-2r=1-5$$

$$-2r=-4$$

$$r=2$$

put $r=2$ in ①

$$= \binom{5}{2} (2)^{5-2} (-1)^2 x^{5-2(2)}$$

$$= \binom{5}{2} (2)^3 (-1)^2 x^{5-4}$$

$$= \frac{5 \times 4}{2 \times 1} \times 8 (1) x^1$$

$$= 80 x$$

$$\therefore \text{coefficient of } x = 80$$

ii) In order to find the coefficient of x in $(1+3x^2)(2x-\frac{1}{x})^5$, we need to find the term of $\frac{1}{x}$ as well $\because 3x^2 \times \frac{1}{x} = 3x$.

From part (i), for term containing $\frac{1}{x}$ we have

$$x^{5-2r} = x^{-1}$$

$$\therefore \frac{1}{x} = x^{-1}$$

$$5 - 2x = -1$$

$$-2x = -1 - 5$$

$$-2x = -6$$

$$x = 3$$

put in ① of part (i)

$$= \binom{5}{3} (2)^{5-3} (-1)^3 x^{5-2(3)}$$

$$= \binom{5}{3} (2)^2 (-1)^3 x^{5-6}$$

$$= \frac{5 \times 4 \times 3}{3 \times 2 \times 1} x^4 (-1) x^{-1}$$

$$= -\frac{40}{x}$$

now from $(1+3x^2)(2x-\frac{1}{x})^5$

$$= (1+3x^2) (80x - \frac{40}{x})$$

$$= 80x - 120x$$

$$= -40x$$

$$\therefore \text{coefficient of } x = -40$$

