Cambridge International AS & A Level

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Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 5-Trigonometry

Question No (31)

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**(i)** Prove the identity $\dfrac{1+\cos\theta}{1-\cos\theta} - \dfrac{1-\cos\theta}{1+\cos\theta} \equiv \dfrac{4}{\sin\theta\tan\theta}$.

**(ii)** Hence solve, for $0° < \theta < 360°$, the equation

$$\sin\theta\left(\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta}\right) = 3.$$

**Solution**

On Next page

(i)

L.H.S $\dfrac{1+\cos\theta}{1-\cos\theta} - \dfrac{1-\cos\theta}{1+\cos\theta}$

$$\frac{(1+\cos\theta)^2 - (1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}$$

$$\frac{(1)^2 + 2(1)(\cos\theta) + (\cos\theta)^2 - ((1)^2 - 2(1)\cos\theta + (\cos\theta)^2)}{1-\cos^2\theta}$$

$$\frac{1+2\cos\theta + \cos^2\theta - (1 - 2\cos\theta + \cos^2\theta)}{1-\cos^2\theta}$$

$$\frac{1+2\cos\theta + \cos^2\theta - 1 + 2\cos\theta - \cos^2\theta}{\sin^2\theta}$$

$$\frac{4\cos\theta}{\sin^2\theta}$$

$$\frac{4}{\frac{\sin^2\theta}{\cos\theta}}$$

$$\frac{4}{\sin\theta\left(\frac{\sin\theta}{\cos\theta}\right)}$$

$$\frac{4}{\sin\theta\tan\theta} = R.H.S$$

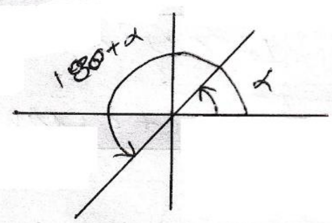
(ii) $\sin\theta\left(\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta}\right) = 3 \quad 0 \le \theta < 360$

$\sin\theta\left(\frac{4}{\sin\theta\tan\theta}\right) = 3$ From part(i)

$\frac{4}{\tan\theta} = 3$

$\tan\theta = 4/3$

tan is +ve in Ist and IIIrd quadrant




Basic angle

$\alpha = \tan^{-1}(4/3)$

$\alpha = 53^\circ.1$

$\therefore \theta = \alpha,\ 180+\alpha$

$= 53^\circ.1,\ 180 + 53^\circ.1$

$\theta = 53^\circ.1,\ 233.1^\circ$ ans