

Cambridge International AS & A Level

Mathematics

9709

Paper 1 Pure Mathematics 1

Topic 5-Trigonometry

Question No (22)

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Question No (22)

It is given that $a = \sin \theta - 3 \cos \theta$ and $b = 3 \sin \theta + \cos \theta$, where $0^\circ \leq \theta \leq 360^\circ$.

- (i) Show that $a^2 + b^2$ has a constant value for all values of θ .
- (ii) Find the values of θ for which $2a = b$.

Solution

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(i)

$$a = \sin \theta - 3 \cos \theta \quad \text{and}$$

$$b = 3 \sin \theta + \cos \theta$$

Now

$$(a)^2 + (b)^2 = (\sin \theta - 3 \cos \theta)^2 + (3 \sin \theta + \cos \theta)^2$$

$$= \sin^2 \theta + 9 \cos^2 \theta - 6 \sin \theta \cos \theta + 9 \sin^2 \theta + \cos^2 \theta + 6 \sin \theta \cos \theta$$

$$= 10 \sin^2 \theta + 10 \cos^2 \theta$$

$$= 10 (\sin^2 \theta + \cos^2 \theta)$$

$$= 10$$

$$\checkmark \sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow a^2 + b^2 = 10 \quad \text{is a constant value.}$$

(ii)

Given condition

$$2a = b$$

putting values of a and b

$$2(\sin \theta - 3 \cos \theta) = 3 \sin \theta + \cos \theta$$

$$2 \sin \theta - 6 \cos \theta = 3 \sin \theta + \cos \theta$$

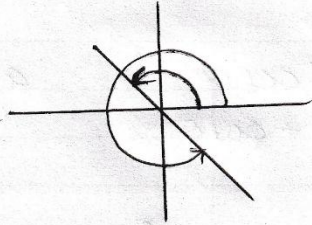
$$2 \sin \theta - 3 \sin \theta = 6 \cos \theta + \cos \theta$$

$$- \sin \theta = 7 \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = -7$$

$$\tan \theta = -7$$

$\tan$ is $-ve$ in II and IV quadrant



basic angle

$$\alpha = \tan^{-1}(7)$$

$$\alpha = 81.9^\circ$$

$$\therefore \theta = 180 - \alpha, 360 - \alpha \quad \text{where } 0 \leq \theta < 360^\circ$$

$$= 180 - 81.9^\circ, 360 - 81.9^\circ$$

$$\theta = 98.1^\circ, 278.1^\circ$$