Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 5-Trigonometry

Question No (11)

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Prove the identity $\frac{\sin x}{1-\sin x} - \frac{\sin x}{1+\sin x} \equiv 2\tan^2 x.$

Solution

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L.H.S $\quad \frac{\sin x}{1-\sin x} - \frac{\sin x}{1+\sin x}$

$$\frac{\sin x(1+\sin x) - \sin x(1-\sin x)}{(1-\sin x)(1+\sin x)}$$

$$\frac{\sin x + \sin^2 x - \sin x + \sin^2 x}{1-\sin^2 x}$$

$\because (a-b)(a+b) = a^2-b^2$

$$\frac{2\sin^2 x}{1-\sin^2 x}$$

$$\frac{2\sin^2 x}{\cos^2 x}$$

$\because \sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$$2\tan^2 x$$