

Cambridge International AS & A Level

Mathematics

9709

Paper 1 Pure Mathematics 1

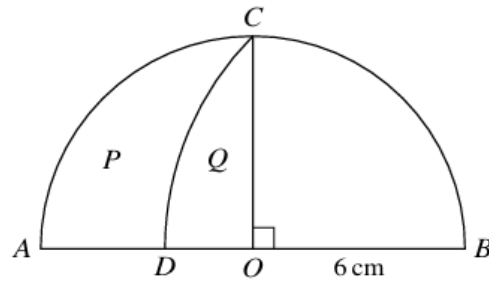
Topic 4-Circular Measure

Question No (21)

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Question No (21)

The diagram shows a semicircle with centre O and radius 6 cm. The radius OC is perpendicular to the diameter AB . The point D lies on AB , and DC is an arc of a circle with centre B .

- (i) Calculate the length of the arc DC .
- (ii) Find the value of
- $$\frac{\text{area of region } P}{\text{area of region } Q},$$
- giving your answer correct to 3 significant figures.

Solution

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(i)

in $\triangle OBC$

$$OB = 6 \text{ cm}$$

$$\text{As } OB = OC$$

$$\Rightarrow \angle CBO = 74^\circ$$

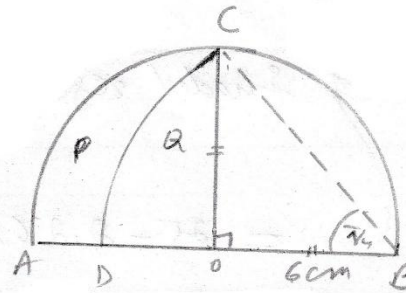
using the Pythagoras

$$BC = \sqrt{OB^2 + OC^2}$$

$$= \sqrt{(6)^2 + (6)^2}$$

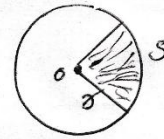
$$= \sqrt{36 + 36}$$

$$BC = \sqrt{72}$$



Arc length formula

$$s = r\theta$$



$$\text{Arc DC} = r\theta$$

$$= \sqrt{72} (74^\circ)$$

$$= \sqrt{2 \times 2 \times 2 \times 3 \times 3} \times 74^\circ$$

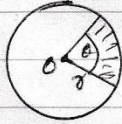
$$= 6\sqrt{2} \times 74^\circ$$

$$\text{Arc DC} = \frac{3\sqrt{2}}{2} \pi \text{ cm}$$

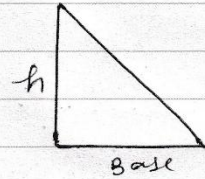
(ii)

Area of sector

$$A = \frac{1}{2} r^2 \theta$$



Area of Triangle



$$\Delta = \frac{1}{2} \text{base} \times h$$

$$\begin{aligned} \text{Area of region Q} &= \text{Area of sector BCD} - \text{area of } \triangle BOC \\ &= \frac{1}{2} r^2 \theta - \frac{1}{2} \text{base} \times \text{height} \\ &= \frac{1}{2} (\sqrt{72})^2 \left(\frac{\pi}{4} \right) - \frac{1}{2} \times 6 \times 6 \\ &= \frac{1}{2} \times 72 \times \frac{\pi}{4} - \frac{36}{2} \\ &= (9\pi - 18) \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of region P} &= \text{Area of quarter circle AOC} - \text{area of region Q} \\ &= \frac{1}{4} (\pi r^2) - (9\pi - 18) \\ &= \frac{1}{4} \pi (6)^2 - (9\pi - 18) \\ &= \frac{36\pi}{4} - (9\pi - 18) \\ &= 9\pi - 9\pi + 18 \\ &= 18 \text{ cm}^2 \end{aligned}$$

$$\therefore \frac{\text{Area of region P}}{\text{Area of region Q}} = \frac{18}{9\pi - 18}$$

$$= \frac{18}{9(\pi - 2)}$$

$$= \frac{2}{\pi - 2} \text{ answer.}$$