Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 4-Circular Measure

Question No (2)

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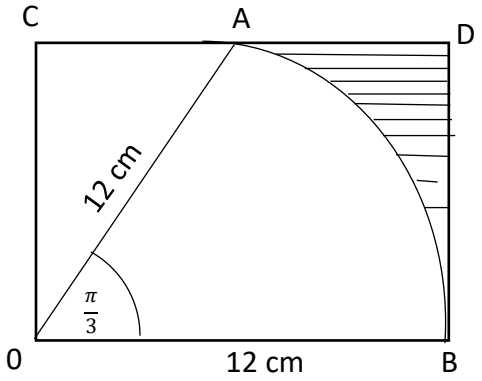

In the diagram, AOB is a sector of a circle with center O and radius 12cm. The point A lies on the side CD of the rectangle OCDB. Angle AOB = $\frac{\pi}{3}$ radians. Express the area of the shaded region in the form a (√3) −bπ, stating the values of the integers a and b.

Solution

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From the diagram

$A\hat{O}C = B\hat{O}C - A\hat{O}B$

$= \frac{\pi}{2} - \frac{\pi}{3}$

$A\hat{O}C = \frac{3\pi - 2\pi}{6} = \pi/6$

From ΔOAC

$\frac{AC}{OA} = \sin A\hat{O}C$

$\frac{AC}{12} = \sin \pi/6$ $\quad \because OA = 12$

$AC = 12 \times \frac{1}{2} = 6$

$\Rightarrow AC = AD = 6$

NOW

$\frac{OC}{OA} = \cos \pi/6$

$OC = OA\left(\frac{\sqrt{3}}{2}\right)$

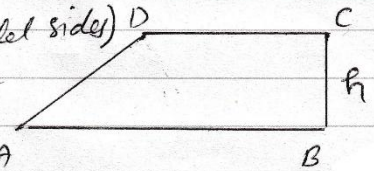
$= 12 \times \frac{\sqrt{3}}{2}$ $\quad \because OA = 12$

$OC = 6\sqrt{3}$

Formula for area of Trapezium

$Area = \frac{1}{2}$ (Sum of parallel sides) × height

$= \frac{1}{2}(|DC| + |AB|) \times h$



Area of Shaded region = Area of Trapezium OAOB − area of sector OAB

$$= \frac{1}{2}(|AD| + |OB|)\,OC - \frac{1}{2}(12)^2\left(\frac{\pi}{3}\right)$$

$$= \frac{1}{2}(6+12) \times 6\sqrt{3} - \frac{1}{2} \times 144\left(\frac{\pi}{3}\right)$$

$$= \frac{18}{2} \times 6\sqrt{3} - 24\pi$$

$$= 54\sqrt{3} - 24\pi$$

So $a = 54$, $b = 24$