Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 4-Circular Measure

Question No (1)

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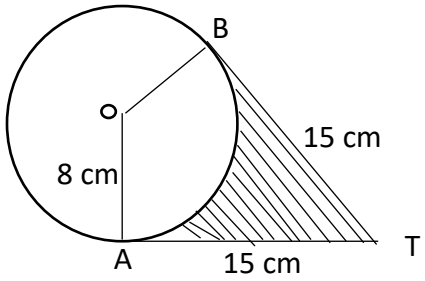

The diagram shows a circle with center O and radius 8 cm. Points A and B lie on the circle. The tangents at A and B meet at the point T, and AT = BT = 15 cm.

(i) Show that angle AOB is 2.16 radians, correct to 3 significant figures.

(ii) Find the perimeter of the shaded region.

(iii) Find the area of the shaded region.

Solution

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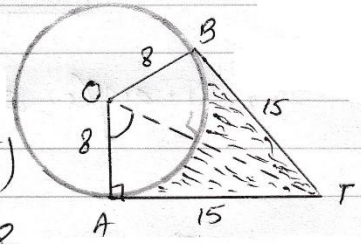
(i) FROM $\triangle OAT$

$\tan A\hat{O}T = \frac{\text{vertical}}{\text{Base}}$

$= \frac{15}{8}$

$A\hat{O}T = \tan^{-1}\left(\frac{15}{8}\right)$

$= 1.0808$



AS $A\hat{O}B = 2(A\hat{O}T)$

$= 2(1.0808)$

$= 2.1616$

$= 2.16$ (3 significant figure)

(ii)

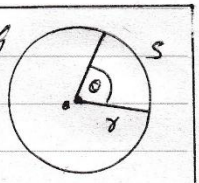
$\overset{\frown}{AB} = r\theta$

$= 8(2.16)$

$\overset{\frown}{AB} = 17.28$

Formula for arc length

$s = r\theta$



∴ perimeter of shaded region $= \overset{\frown}{AB} + BT + AT$

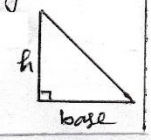
$= 17.28 + 15 + 15$

$= 47.3$

(iii)

Formula for area of triangle

$\Delta = \frac{1}{2}$ base × height



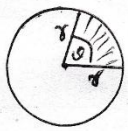
Area of triangle $OAT = \frac{1}{2} \times 8 \times 15$

$= 60 \text{ cm}^2$

Area of triangle $OBT = 60 \text{ cm}^2$

Formula for area of sector

$A = \frac{1}{2} r^2 \theta$



Area of sector $AOB = \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(8)^2 (2 \cdot 16)$ $\quad \therefore \theta = 2.16$ part (i)

$= 69 \cdot 12 \text{ cm}^2$

∴ area of shaded region

= Area of triangle OAT + Area of Triangle OBT − Area of sector AOB

$= 60 + 60 - 69 \cdot 12$

$= 50 \cdot 9 \text{ cm}^2$