

Cambridge International AS & A Level

Mathematics

9709/32

Paper 4 Mechanics

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Question No(2)

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A particle P moves in a straight line. At time t s after leaving a point O on the line, P has velocity $v \text{ m s}^{-1}$, where $v = 44t - 6t^2 - 36$.

- (a) Find the set of values of t for which the acceleration of the particle is positive.
 (b) Find the two values of t at which P returns to O.

Solution:

Given

$$v = 44t - 6t^2 - 36$$

$$\frac{dv}{dt} = 44 - 6(2t) - 0$$

$$\frac{dv}{dt} = 44 - 12t$$

acceleration is +ve when

$$\frac{dv}{dt} > 0$$

$$\Rightarrow 44 - 12t > 0$$

$$-12t > -44$$

$$12t < 44$$

$$t < \frac{44}{12}$$

$$t < \frac{11}{3}$$

So acceleration will be +ve for

$$t < \frac{11}{3}$$

(b)

Given

$$v = 44t - 6t^2 - 36$$

$$\frac{ds}{dt} = 44t - 6t^2 - 36 \quad \because v = \frac{ds}{dt}$$

$$ds = (44t - 6t^2 - 36) dt$$

$$\int ds = \int (44t - 6t^2 - 36) dt$$

$$s = \frac{44t^{2+1}}{2+1} - 6 \frac{t^{3+1}}{3+1} - 36t + C \rightarrow (1)$$

C is constant of integration

$$\text{At } t=0, s=0$$

$$\Rightarrow C=0$$

$$\Rightarrow s(t) = \frac{44t^2}{2} - \frac{6t^3}{3} - 36t$$

$$s(t) = 22t^2 - 2t^3 - 36t$$

particle return to 0, mean

$$s(t) = 0$$

$$\Rightarrow 22t^2 - 2t^3 - 36t = 0$$

$$2t(11t - t^2 - 18) = 0$$

$$2t = 0, \quad 11t - t^2 - 18 = 0$$

$$t = 0,$$

$$11t - t^2 - 18 = 0$$

$$-t^2 + 11t - 18 = 0$$

$$-1(t^2 - 11t + 18) = 0$$

$$t^2 - 11t + 18 = 0$$

factorize

$$t^2 - 9t - 2t + 18 = 0$$

$$t(t-9) - 2(t-9) = 0$$

$$(t-9)(t-2) = 0$$

$$t-2 = 0, \quad t-9 = 0$$

$$t = 2, \quad t = 9$$

So, p will return to 0, at $t = 2, 9$