

Cambridge International AS & A Level

Mathematics

9709/12

Paper 1 Pure Mathematics 1

May/June 2024

Question No (1)

<http://kingcambridgesolutions.com>

Question No (1)

The coefficient of x^2 in the expansion of $(1 - 4x)^6$ is 12 times the coefficient of x^2 in the expansion of $(2 + ax)^5$.

Find the value of the positive constant a .

Solution:

Binomial Expansion

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots$$

Expansion of $(1-4x)^6$ up to x^2

$$(1-4x)^6 = 1 + 6(-4x) + \frac{6(6-1)}{2!}(-4x)^2 + \dots$$

$$= 1 - 24x + 240x^2 + \dots$$

coefficient of $x^2 = 240$

Now for the expansion of $(2+ax)^5$ up to x^2

$$(2+ax)^5 = 2^5 \left(1 + \frac{a}{2}x\right)^5$$

$$= 32 \left(1 + \frac{a}{2}x\right)^5$$

using expansion

$$(2+ax)^5 = 32 \left[1 + 5\left(\frac{a}{2}\right)x + \frac{5(5-1)}{2!} \left(\frac{a}{2}x\right)^2 + \dots \right]$$

$$= 32 \left[1 + \frac{5}{2}ax + \frac{5 \times 4}{2 \times 1} \times \frac{a^2}{4}x^2 + \dots \right]$$

$$= 32 + 80ax + \frac{32 \times 5 \times 4}{2 \times 1} \times \frac{a^2}{4}x^2 + \dots$$

$$= 32 + 80ax + 80a^2x^2 + \dots$$

coefficient of x^2 , $80a^2$

According to the given condition

coefficient of x^2 for $(1-4x)^6 = 12$ (coefficient
of x^2 for $(2+ax)^5$)

$$\Rightarrow 240 = 12(80a^2)$$

$$240 = 960a^2$$

$$a^2 = \frac{240}{960}$$

$$a^2 = \frac{24}{96}$$

$$a = \sqrt{\frac{24}{96}}$$

$$a = 0.5$$

$\therefore a$ is +ve